EECS 16A Imaging 3

We will start at Berkeley Time!

Last time: Matrix-vector multiplication

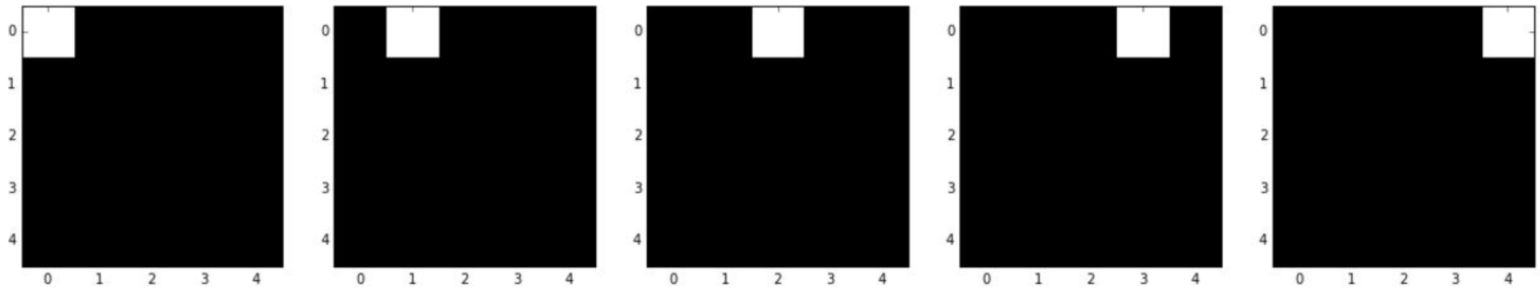
$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \cdots \\
 & & & & \cdots & & & &
\end{bmatrix}
\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \\ \\ \\ i_n \end{bmatrix}
=
\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ \\ \\ \\ s_n \end{bmatrix}
$$

Masking Matrix H

Unknown, vectorized image, $\vec{i}$

Recorded Sensor readings, $\vec{s}$

Last time: Single-pixel scanning



- Setup a masking matrix where each row is a mask
  - Measured each pixel individually once

$$\vec{s} = H\vec{i}$$

- How did we reconstruct our image, once we had s?

Poll Time! (this is review)

What are the requirements of our masking matrix H? (multiple choice)

A. H is invertible
B. H has linearly independent columns
C. H has a trivial nullspace
D. Determinant of H is 0
E. Unique solution to Hi = s

$$\vec{s} = H\vec{\imath}$$

Our system

Poll Time! (this is review)

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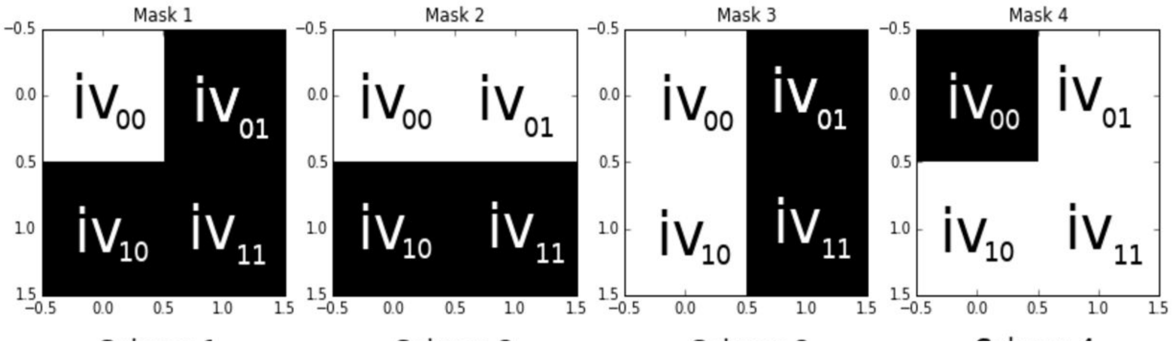
Our system

Questions from Imaging 2

Goal: Understand which measurements are good measurements

- ✓ Can we always reconstruct our image? **Need invertible H**
- ? Are all invertible matrices equally good as scanning matrices?
- ? What happens if we mess up a single scan?
- ? What if we use multiple pixel instead of single pixel scan?

Today: Multipixel scanning



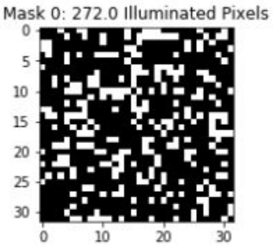
- **Can we measure multiple pixels at a time?**
  - Measurements are now linear combinations of pixels

- **How can we reconstruct our scanned image?**
  - Can multipixel masks still be linearly independent, aka invertible?

Why do we care?

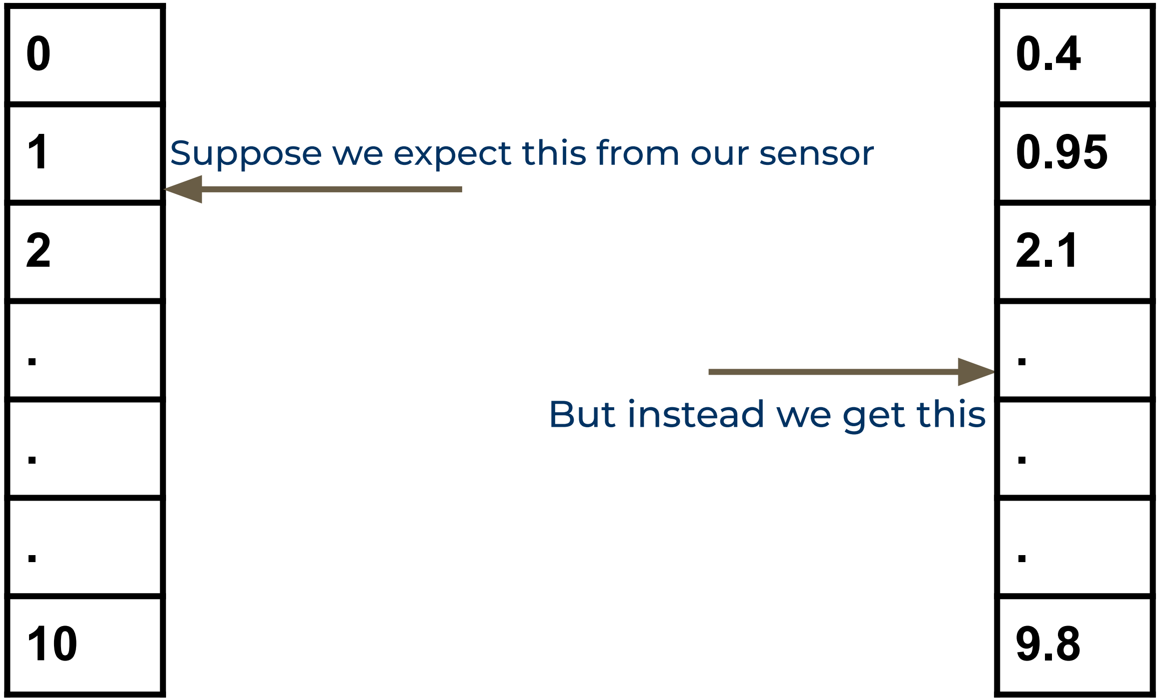
- Improve image quality by **redundancy**:
    - Ideally, one measurement is enough
    - Redundancy: conducting more measurements
    - Extract pixel value by averaging over multiple measurements
        - Good measurements → good average
        - Occasional bad measurements → worsen the pixel value but makes it tolerant of some errors

How do we do it?



- Illuminate multiple pixels per scan:
    - Each mask measures a linear combination of pixels instead of a single pixel, i.e. has multiple 1's
    - Increases signal level:
        - Signal is data that we do want (ex: light intensity from pixel illumination)
- Problems:
    - Our measurements are noisy
        - **Noise** is a random unwanted variation in our measurement (ex: room light getting into box)
        - Noise may be amplified through inverting a matrix!
- **Goal: high signal, low noise →** high signal-to-noise ratio (SNR)

What is noise?



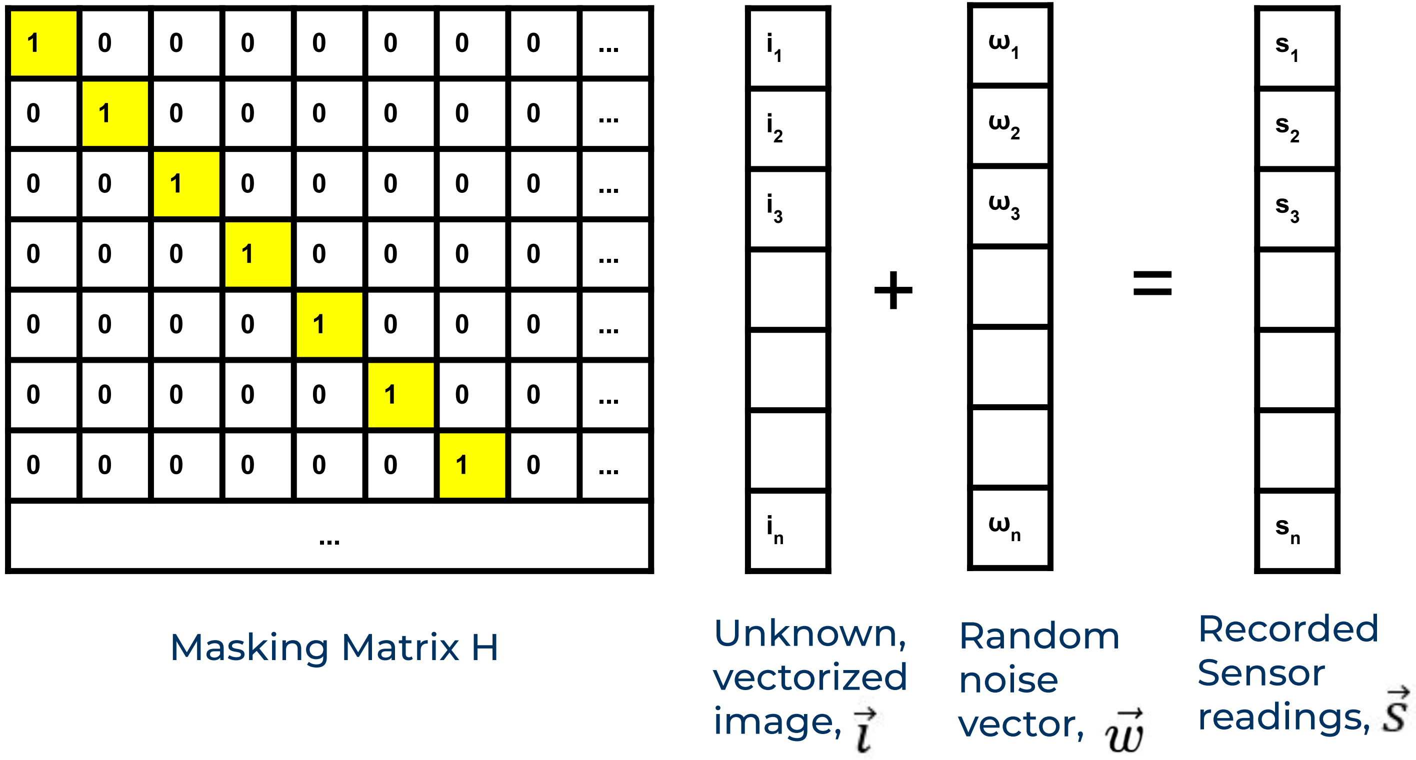
What is noise?

Measured values =
ideal vector + noise vector (ω)

$$\begin{bmatrix} 0.4 \\ 0.95 \\ 2.1 \\ . \\ . \\ . \\ 9.8 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ . \\ . \\ . \\ 10 \end{bmatrix} + \begin{bmatrix} 0.4 \\ -0.05 \\ 0.1 \\ . \\ . \\ . \\ -0.2 \end{bmatrix}$$

$$\vec{s}_{real} \qquad \vec{s}_{ideal} \qquad \vec{w}$$

How does noise affect our system?



A more realistic system

- Sensor readings = H applied to image vector + noise vector

$$\vec{s} = H\vec{i} + \vec{w}$$

- We can't reconstruct exact **i**, but we can estimate it

$$\vec{i}_{est} = H^{-1}\vec{s} = \vec{i} + \boxed{H^{-1}\vec{w}}$$

Be careful about the <u>reconstruction error</u> (noise term) or else it could blow up !!

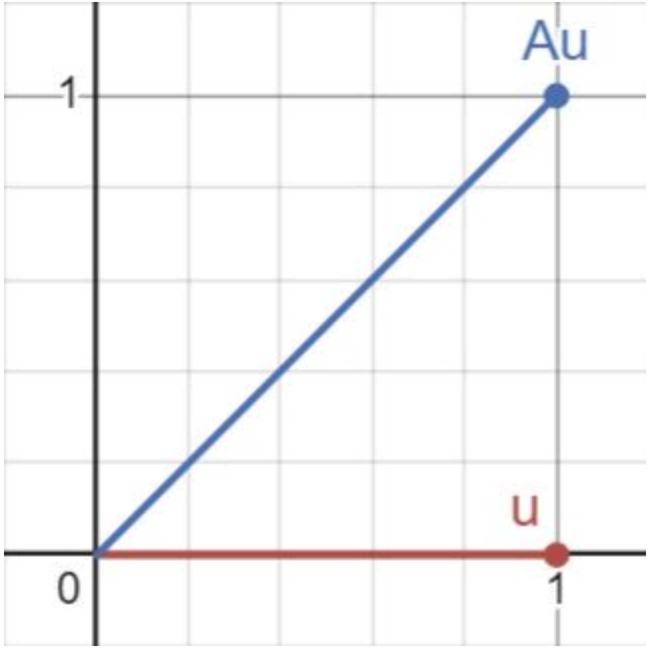
Eigenvectors! (and Eigenvalues)

First, regular matrix-vector multiplication

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Au = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



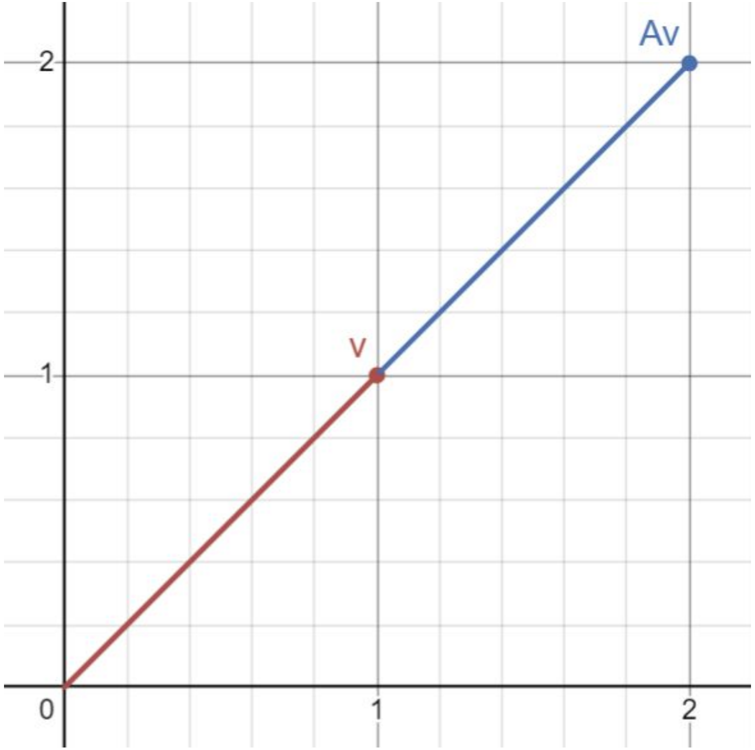
Eigenvectors and Eigenvalues

What is an eigenvector?

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$Av = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

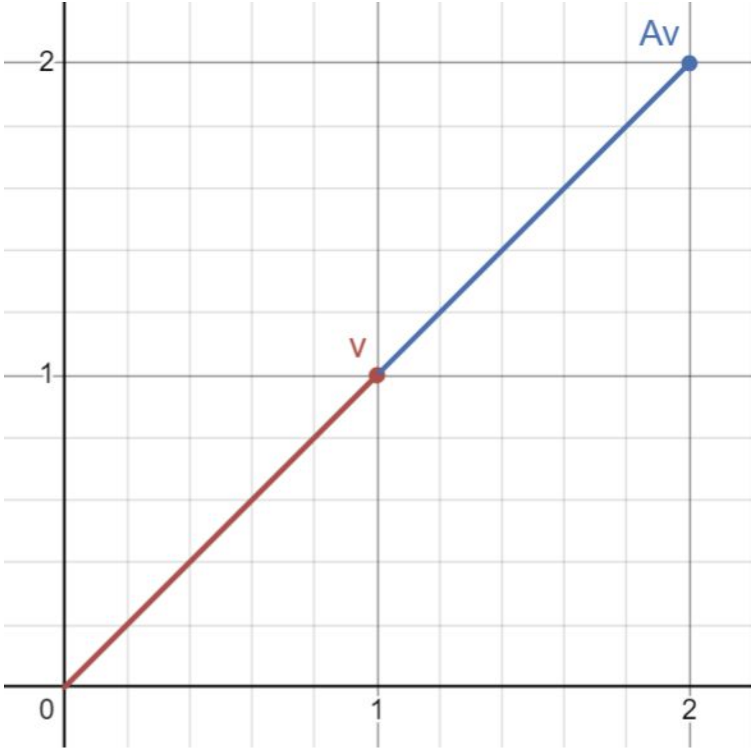


Eigenvectors and Eigenvalues

- Av and v are on the same line
  - Thus v is an eigenvector of A
- Another way of saying this:
  - Av is a scalar multiple of v, specifically, Av = 2v
  - Thus v's eigenvalue is 2
  - A has eigenpair (v, 2)

In general, v is an eigenvector of A with eigenvalue λ when

$$Av = \lambda v$$



Eigenvalues of Invertible Matrices

- All invertible matrices do not have a 0 eigenvalue.

Why?

- Consider some matrix that has a eigenvalue of 0.

$$Av = 0v = 0$$

- We see A has a non-trivial nullspace and thus it's not invertible
- We've proven what we wanted to prove!

Back to our scanning system with noise

- Sensor readings = H applied to image vector + noise vector

$$\vec{s} = H\vec{i} + \vec{w}$$

- We can't reconstruct exact **i**, but we can estimate it

$$\vec{i}_{est} = H^{-1}\vec{s} = \vec{i} + \boxed{H^{-1}\vec{w}}$$

Be careful about the reconstruction error (noise term) or else it could blow up !!

Eigenvalues for inverse matrices

- H is an invertible NxN matrix ↔ trivial nullspace
  - No zero eigenvalues
- Assume H has N linearly independent eigenvectors
  - $\mathrm{H}v_i = \lambda_i v_i \text{ for } i = 1 \ldots \mathrm{N}$
- The eigenvectors span $\mathbb{R}^N$ → where the noise vectors "live"!
- Eigenvalue-eigenvector definition:

$$\mathrm{H}^{-1}v_i = \frac{1}{\lambda_i}v_i \text{ for } i = 1 \ldots \mathrm{N}$$

Proof:

$$H\vec{v_i} = \lambda_i \vec{v_i}$$

$$\implies H^{-1}H\vec{v_i} = \lambda_i H^{-1}\vec{v_i}$$

$$\implies H^{-1}\vec{v_i} = \frac{1}{\lambda_i}\vec{v_i}$$

How do eigenvalues affect noise?

The noise vector can be written as a linear combination of eigenvectors:

$$\vec{\omega} = \alpha_1\overrightarrow{v_1} + \alpha_2\overrightarrow{v_2} + \cdots \alpha_n\overrightarrow{v_n}$$

Including effect of H^{-1}

$$H^{-1}\vec{\omega} = H^{-1}(\alpha_1\overrightarrow{v_1} + \alpha_2\overrightarrow{v_2} + \cdots \alpha_n\overrightarrow{v_n})$$

Rewritten with eigenvalues:

$$H^{-1}\vec{\omega} = \frac{1}{\lambda_1}\alpha_1\overrightarrow{v_1} + \frac{1}{\lambda_2}\alpha_2\overrightarrow{v_2} + \cdots \frac{1}{\lambda_n}\alpha_n\overrightarrow{v_n}$$

Linking it all together

$$\vec{i}_{est} = H^{-1}\vec{s} = \vec{i} + \boxed{H^{-1}\vec{w}}$$

$$\boxed{H^{-1}\vec{\omega}} = \frac{1}{\lambda_1}\alpha_1\overrightarrow{v_1} + \frac{1}{\lambda_2}\alpha_2\overrightarrow{v_2} + \cdots \frac{1}{\lambda_n}\alpha_n\overrightarrow{v_n}$$

- Remember: want small noise term for high signal-to-noise ratio
- The noise is directly related to the eigenvalues

Poll Time!

- Do we want small or large eigenvalues for the H matrix in order to get a good image?
  A. Large
  B. The magnitude doesn't matter
  C. Small

- Which of the following equations correctly model our imaging system? (multiple choice)
  A. $s_{ideal} = H.i$
  B. $s_{real} = s_{ideal} + w = H.i + w$
  C. $s_{real} = s_{ideal} + w = H.i + H.w$
  D. $i_{est} = H^{-1}.s_{real} = H^{-1}.s_{ideal} + H^{-1}.w$
  E. $i_{est} = H^{-1}.s_{real} = H^{-1}.s_{ideal} + w$

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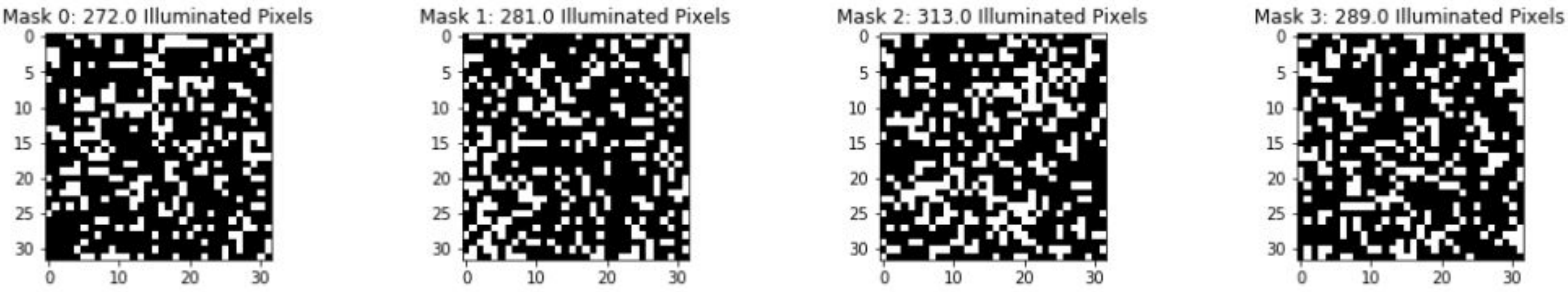
Possible scanning matrix: Random

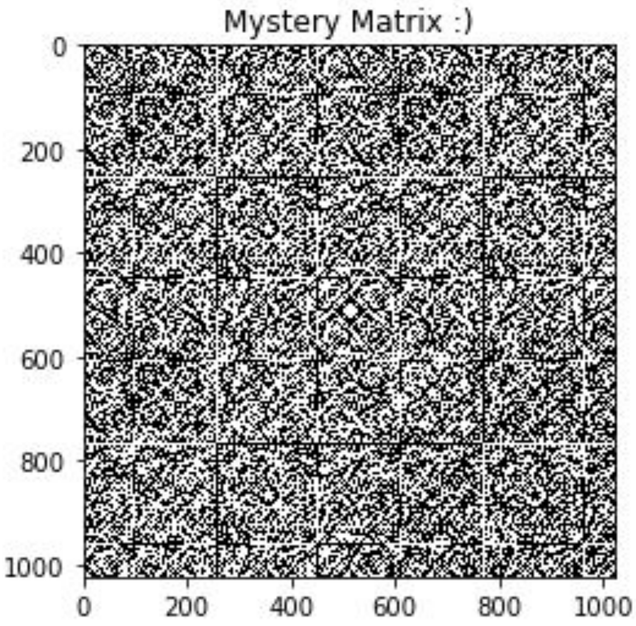


- Illuminate ~300 pixels per scan
    - *Usually* invertible
    - But what are its eigenvalues?

¯_(ツ)_/¯

A more systematic scanning matrix

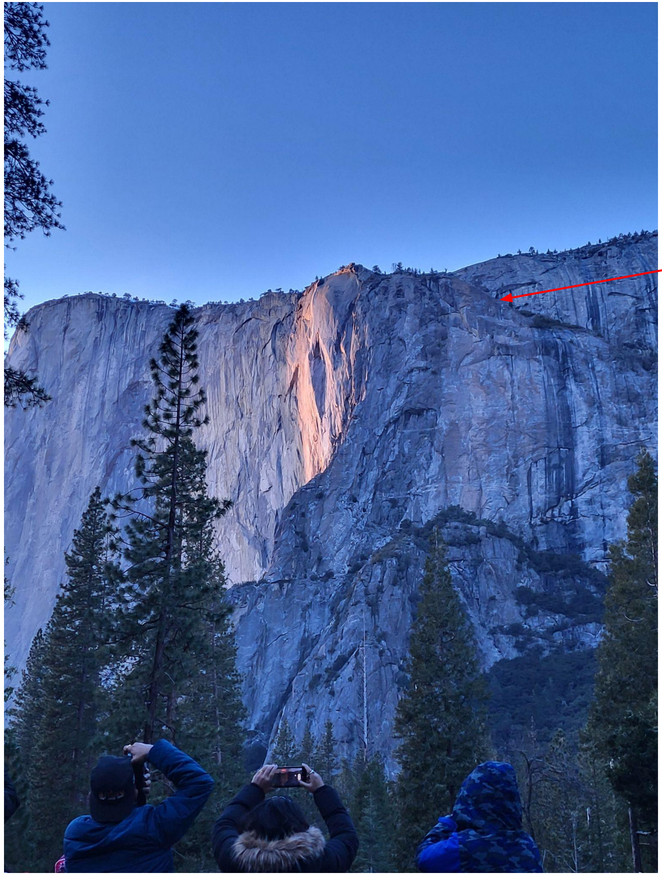
- Hadamard matrix!
- Constructed to have large eigenvalues
  - Just what we need!



Multipixel Scanning Use Cases

- Not the "superior technology" – as any practice, it has its advantages and disadvantages:
  - Multipixel scan is useful if we cared about getting *close* to each pixel value, prioritizing getting decent results for each pixel
  - Single-pixel scan allows to get really good measurements for some pixels while other pixels will be lost entirely

Example: Horsetail Falls



Do you see the missing pixel?

Example: Horsetail Falls Comparison

Single-pixel: many “perfect” pixels but some are entirely missing

Multipixel: fewer “perfect” pixels (creating blurriness) but information-preserving

Multipixel Scanning Expectations

- Note: when shining light at multiple portions of an image, our light will easily bleed to the pixels around one region
  - *Therefore, we may not get better results*
- **Today's goal:**
  - Show that it's possible to get results using multipixel scanning
  - **Use case of multipixel scanning at the end of the lab**

Workflow and Debugging

- READ CAREFULLY
- Circuit:
  - Resistor is different
  - Check light sensor orientation
  - **Red** jumper for **+**
  - **Black** jumper for **GND**
- Projector:
  - **BE CAREFUL WITH PORTS (DC, HDMI)**
  - brightness 0, contrast 100
  - may restart in the middle of scan
- Image: not-too-detailed
- Cover box with jacket for dark conditions
- Project masks onto image
- Reconstruct image: $H^{-1}\mathbf{s}_{\mathbf{real}} = \mathbf{i}_{\mathbf{est}}$